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Engineering Note

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Thermally Induced Attitude Disturbance Control for Spacecraft with a Flexible Boom

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Introduction

A SPACECRAFT is made up of a primary structure, to which are attached appendages such as gravity-gradient booms, antennae, solar arrays, radiators, scientific instruments, and other payloads. It is necessary to design lighter space structures to save launch costs. This kind of design strategy results in space structures being flexible with very low-frequency fundamental vibration modes. For many years, spacecraft appendages exposed to solar radiation have shown thermally induced structural deformations.[1] The deformations can be transferred into structural vibrations or steady-state bending, or both. These structural disturbances have resulted in attitude disturbances of spacecraft that can be larger than required attitude pointing accuracy. Hence, sometimes active controls of attitude disturbances should be performed to maintain a pointing accuracy of spacecraft attitude.

Previous research on these kind of thermal effects has focused on structural response based on temperature differences.[1] Recently, attitude dynamics have been included for a spacecraft incorporating the effects of a flexible boom undergoing thermal loading.[2] For slew maneuvers or vibration control of spacecraft, many authors have already solved hub–appendage problems to study the coupled dynamics of flexible spacecraft.[3,4] Combing a coupled hub–appendage system with thermally induced attitude model can yield a sophisticated model to solve the attitude dynamics of spacecraft with a boom suffering from thermal effects. One goal of this Note is to refine the model of the attitude dynamics first studied in Ref. 2 by using the finite element method (FEM). This refined model also includes a rotatory inertia of tip-mass, models of a sensor and an actuator attached the flexible boom, and the moment applied by control torques, as well as the moment produced by thermal loading. The main objective of this study is to develop a new control algorithm for suppression of the attitude disturbances due to thermal effects on a flexible spacecraft appendage by using piezoceramic materials. The new control algorithm must be incorporated with the attitude dynamics and should handle both the steady-state errors and vibrations because the attitude disturbances have both of them. Numerical studies are performed to analyze the characteristics of the new control method for a spacecraft with a realistic boom.

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Spacecraft Modeling

Equations of Motion

An accurate model of a spacecraft system should be developed to analyze the dynamic response of the system and the effect of an applied control law. After achieving the mathematical modeling, we can develop a control law based on the desired control objective. This Note considers a spacecraft that consists of a rigid hub and a cantilevered flexible boom, with a tip mass as shown in Fig. 1. The coupled dynamics of rigid hub and flexible boom are performed to consider interactions between the rigid-body motions θ of the spacecraft and the dynamic motions y of the boom. The governing equations of motion and boundary conditions for the system can be derived from a generalized form of Lagrange's equation.[3,4] To include the thermal torque M_T and control moment M_P in the governing equations, we can use Eq. (1), which describes the work done by external torques and controls:

$$W = \int_{l_0}^{l} \left(M_T \frac{\partial^2 y}{\partial x^2} + M_P \frac{\partial^2 y}{\partial x^2} \right) \mathrm{d}x \tag{1}$$

where x is auxiliary variable measured from the outer radius of the hub along the undeformed boom axis; y is displacement from x axis; l is length of boom, 7.5 m; and l_0 is radius of hub, 1.0 m. The equation of rotation motion for rigid hub and the governing equation of the boom deformation in the body-fixed reference coordinate are obtained as, respectively,

$$I_{\text{hub}}\ddot{\theta} + \int_{l_0}^{l} \rho A x(\ddot{y} + x\ddot{\theta})\,\mathrm{d}x + m_{\text{tip}} l(l\ddot{\theta} + \ddot{y}|_l) + I_{\text{tip}}\left(\ddot{\theta} + \left.\frac{\partial \ddot{y}}{\partial x}\right|_l\right) = 0 \tag{2}$$

$$\rho A x\ddot{\theta} + \rho A\ddot{y} + EI\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 (M_T + M_P)}{\partial x^2} = 0 \tag{3}$$

where I_{hub} is moment of inertia of hub, 1250 $\mathrm{kg \cdot m^2}$; θ is attitude rotation angle of hub (spacecraft); ρ is mass density of boom, 8026 $\mathrm{kg/m^3}$; A is cross section area of boom, 1.08×10^{-5} $\mathrm{m^2}$; m_{tip} is tip mass, 1.5 kg; I_{tip} is rotatory inertia of the tip mass, 0.001 $\mathrm{kg \cdot m^2}$; I is moment of inertia of the boom, 4.3523×10^{-10} $\mathrm{kg \cdot m^2}$; E is Young's modulus of the boom, 1.93×10^{11} $\mathrm{N/m^2}$; and the umlauts

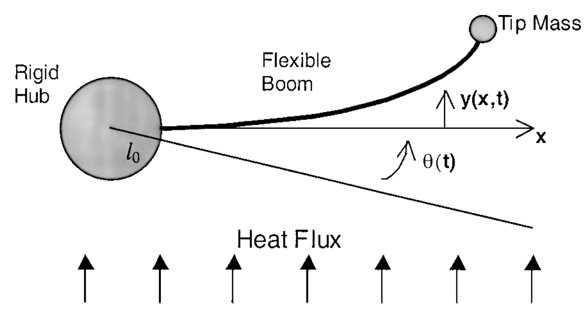

Fig. 1 Spacecraft model with a flexible boom.

signify differentiation with respect to time. At $x = l_0$, the boom is cantilevered so that the essential boundary conditions are, at $x = l_0$,

$$y = 0, \qquad \frac{\partial y}{\partial x} = 0 \tag{4}$$

At $x = l$, because the boom supports the tip mass, the natural boundary conditions of are given by, at $x = l$,

$$\frac{\partial(M_T + M_P)}{\partial x} + EI\frac{\partial^3 y}{\partial x^3} = m_{\text{tip}}(l\ddot{\theta} + \ddot{y}) \tag{5a}$$

$$EI\frac{\partial^2 y}{\partial x^2} + M_T + M_P = -I_{\text{tip}}\left(\ddot{\theta} + \frac{\partial \ddot{y}}{\partial x}\right) \tag{5b}$$

Equation (5a) shows the shear force on the tip mass, whereas Eq. (5b) explains the moment at the tip mass. The rotatory inertia I_{tip} of the tip mass is not neglected, such that the right-hand side of Eq. (5b) is not zero.

Finite Element Application

We know that, in the most common circumstances, configuration admissible functions in the assumed modes method satisfy only the essential boundary, but do not satisfy the natural boundary conditions. The FEM usually gives a fast convergence to the true model with fewer degrees of freedom than the assumed modes method.[3] Thus, we prefer to use a finite element model rather than use a model of assumed modes method used in Ref. 2. The FEM transforms the partial differential equations of motion [Eqs. (2–5b)] into finite dimensional differential equations in terms of displacements, velocities, and accelerations in the FEM coordinates.[3,4] The displacement over a finite element can be discretized using a finite element expansion that consists of the transverse deflection and rotation of the element and Hermite polynomials chosen to satisfy the boundary conditions at both ends of the element. The work done by an applied moment from thermal deformation or an actuator is assumed to be a constant over one element. In this Note, piezoceramic materials are used as an actuator and a sensor that are attached to the flexible boom. The piezoceramic elements used should be incorporated into the finite element. By the use of the electromechanical coupling effect of piezoceramic materials, the potential energy of the piezoceramic element can be obtained.[5] Combining the equation of potential energy with a finite element expansion and integration by parts yield the stiffness matrix of piezoceramic element.[5] The piezoceramic elemental mass matrix can be obtained in the same fashion as an ordinary structural element. Some structural elements of the boom have piezoceramic material bonded to them. The respective stiffness matrix and mass matrix of the structural elements and attached piezoceramic elements are given by the simple addition of the boom element matrices and the piezoceramic elemental matrices. After we derive the equations for every finite element, we can combine all equations for each element into global governing equations. The equations can be written in matrix form as

$$[M]\{\ddot{Q}\} + [K]\{Q\} = F_P M_P + F_T M_T \tag{6}$$

where Q is generalized coordinates for the system, $[M]$ is global mass matrix, $[K]$ is global stiffness matrix, F_P is input influence matrix for control, and F_T is input influence matrix for thermal effects. F_p is dependent on the numbers of actuators and sensors and their locations. Q consists of θ and collections of q_i at each node. The odd number of q, that is, $q_1, q_3, q_5, \ldots$, is the vertical displacement at each node, and the even number of q, that is, $q_2, q_4, q_6, \ldots$, is the rotation at each node. The equations of motion can be decoupled and more easily analyzed by the use of modal coordinates.[6] The transformation of the generalized coordinates into modal coordinates can be performed by using $Q = \Phi\xi$. Φ is the weighted modal matrix and is obtained by solving the eigenvalue problem associated with the $[M]$ and $[K]$ matrices. Applying the modal coordinate to Eq. (6) and premultiplying by Φ^T gives the following uncoupled equation:

$$\{\ddot{\xi}\} + [2\zeta\omega]\{\dot{\xi}\} + [\omega^2]\{\xi\} = \Phi^T F_p M_P + \Phi^T F_T M_T \tag{7}$$

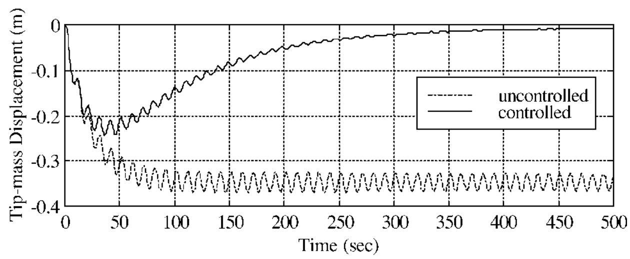

Fig. 2 Tip-mass displacements due to thermal loading and the modified PPF control.

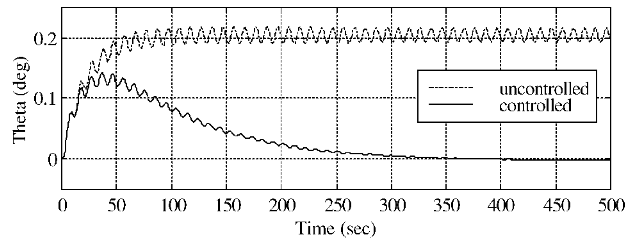

Fig. 3 Attitude angles due to thermal loading and the modified PPF control.

where ξ is the modal coordinate, ζ is the viscous damping constant of the boom, and ω is modal frequency. In Eq. (7), we introduce the damping matrix $[2\zeta\omega]$ because the system is considered to have viscous damping.

Thermally Induced Attitude Dynamics

Temperature differences of spacecraft boom give differential thermal expansion through the boom. When temperature differences change rapidly, the spacecraft may experience dynamic structural motion and vibration. These deformations and vibrations of the flexible boom lead to rigid-body rotation of the spacecraft due to the conservation of the total angular momentum of the spacecraft.[2] The spacecraft chosen for this study has the physical model and flexible boom (a uniform stainless steel) properties described in Ref. 2. Additionally, we include the rotatory inertia of the tip mass in this study because it contributes to the stability of control. The thermal moment of the boom for the thermal-structural response is that used in the description in Ref. 2. With this thermal moment, the spacecraft modeled in the preceding section is numerically simulated. This numerical analysis does not include the formulations of sensor, actuator, and control moments. The boom deformations are subjected to a rapid change in thermal loading and are dominated by only the first structural mode. These numerical results are the same as those in Ref. 2. The dynamic response of the boom, that is, tip mass, consists of a quasi-static displacement and superimposed vibrations, as shown in Fig. 2 for an uncontrolled case. The attitude angle response (see an uncontrolled case in Fig. 3) also consists of a pointing error (steady-state error) and superimposed vibrations in the direction opposite to the boom dynamics. As mentioned in Ref. 2, as the mass of hub becomes small, that is, small spacecraft, the torque resulting from deformations of the boom has a great influence. This influence makes the pointing error and jitter dramatically increase. Thus, for small spacecraft, it is necessary to implement control techniques for suppression of these attitude disturbances.

Development of a New Control Technique

As one of smart structures, piezoeceramic materials generate an electric charge in response to a mechanical deformation, and they also yield mechanical deformation when subjected to an electric voltage. These smart materials are used as either actuators or sensors for a wide variety of applications, such as acoustic attenuation, vibration suppression, and shape control. In this Note, we want to control the structural vibrations and deformations of the flexible boom by providing the voltage developed from the piezoceramic actuators through compensators. The sensors and actuators are collocated at appropriate finite elements of the boom represented in the preceding section. The properties of the piezoceramic used are length of 0.15 m, thickness of 0.0019 m, width of 0.019 m, density of 7700 kg/m^3, Young's modulus of 6.3×10^{10} N/m^3, and lateral charge coefficient of 1.8×10^{-10} m. The voltage produced by the piezoceramic sensors and the moment generated by the piezoceramic actuators can be calculated from the equations in Ref. 5.

A new control algorithm, called positive position feedback (PPF), has been developed for vibration control.[7] PPF means positively feeding a structure coordinate back to a compensator and positively feeding back the compensator coordinate to the structure. PPF has been used as a vibration suppression control strategy because it is not sensitive to spillover and is not destabilized by actuator dynamics.[7] By the use of the PPF control algorithm with piezoceramic sensors and actuators, previous studies have shown a great degree of control when compensators were precisely tuned to the modal frequencies of the structure.[5–7] As mentioned in the preceding section, only the first mode of structural deformation is dominantly exited when the boom is heated. Thus, first, we focus on controlling only the first mode with a collocated piezoceramic actuator/sensor pair. The original PPF enables the piezoceramic actuator to remove only vibrations due to thermal effects on the boom. The steady-state error caused by the bending of the boom cannot be reduced by only the original PPF. Increasing the PPF gain has limits because of saturation and noise effects, although it can eliminate the disturbances. The effects of steady-state disturbances can be reduced by an integral control. Thus, a modified PPF compensator design in this Note adds integral control to the original PPF for both vibration and bending control of flexible space structures. The new method provides extended capability of controlling structural natural frequencies and damping. The modified PPF of the scalar case is obtained as

$$\ddot{\xi} + [2\zeta\omega]\dot{\xi} + [\omega^2]\xi = \left[\Phi^T F_P\right]g\eta - \left[\Phi^T F_P\right]g_q\left[\Phi^T F_P\right]^T \int \xi \,\mathrm{d}t + \Phi^T F_T M_T \tag{8a}$$

$$\ddot{\eta} + [2\zeta_c\omega_c]\dot{\eta} + \left[\omega_c^2\right]\eta = \omega_c^2\left[\Phi^T F_P\right]^T \xi \tag{8b}$$

where g is the feedback gain of PPF, g_q is a new positive definite feedback gain, η is the compensator state, ζ_c is modal damping of the compensator, and ω_c is the compensator modal frequency. Equation (8a) describes the structure, that is, spacecraft, whereas Eq. (8b) describes the compensator. The response, including vibrations and deformations of the boom, is indicated by the modal coordinate ξ. This response produces a voltage on the piezoceramic sensor, which is input to the compensator. The compensator, consisting of a PPF filter, calculates the control voltage. Then, the control voltage is applied to the piezoceramic actuator that produces the control moment M_p. From Eqs. (8a) and (8b), we can understand that the modified PPF is introduced to make use of the integration of output from the position sensor. For a multimode case of the modified PPF with more than one mode and a compensator with more than one filter, the system can be easily extended as

$$\ddot{\boldsymbol{\xi}} + D\dot{\boldsymbol{\xi}} + \Omega\boldsymbol{\xi} = C^T G\boldsymbol{\eta} - C^T G_q C \int \boldsymbol{\xi}\,\mathrm{d}t + \Phi^T F_T M_T \tag{9a}$$

$$\ddot{\boldsymbol{\eta}} + D_c\dot{\boldsymbol{\eta}} + \Omega_c\boldsymbol{\eta} = \Omega_c C\boldsymbol{\xi} \tag{9b}$$

where $\boldsymbol{\xi}$ is a modal state vector of N_1 dimension, $\boldsymbol{\eta}$ is the filter state vector of N_2 dimension, D is the modal damping diagonal matrix of N_1, D_c is the filter damping diagonal matrix of N_2, Ω is the modal frequency diagonal matrix of N_1, Ω_c is the filter frequency diagonal matrix of N_2, G is the diagonal matrix for PPF gains of N_2, G_p is the diagonal matrix for integral gains of N_2, and C is the full participation matrix of $N_2 \times N_1$. C^T is defined as the $\Phi^T F_P$ matrix that is dependent on the number of actuators and sensors and number of modes to control. The addition of integral control while removing steady-state error may lead to an unstable control system. The integral control adds extra complexity to this system so that a stability condition can hardly be derived by an analytical method.

Numerical Applications and Results

To suppress the attitude disturbances due to thermal effects, control moments can be applied from piezoceramic actuators using the control law of the modified PPF. The state equations of the system are written in vector matrix form directly from Eqs. (9a) and (9b), and the overall system can be easily solved by a numerical tool. The behavior of this system is dependent on the characteristics of the flexible boom and the spacecraft. Like the original PPF, the modified PPF control system provides the effective control when the compensator is tuned to the modal frequency. Because of the strong coupling effects caused by integral control, the multimode system cannot be approximated by the N_1 decoupled modal subsystem. To first-order accuracy, the dynamics of a bundled subsystem, consisting of only the modes to be controlled, can be approximately considered as the dynamics of the whole system. The first modal frequency dominantly contributes to boom deformations, the second mode has a little contribution, and the rest of modes have a negligible contribution. When attitude disturbance θ due to each mode is checked, attitude disturbance due to the second mode is about a thousand times less than the disturbance due to the first mode. Thus, it is a good idea that only one actuator with one compensator is used to suppress attitude disturbances caused by the first mode excitations. When only the first mode is used to build an approximated whole system, the characteristic equation of the augmented closed-loop system can be obtained by using the modified PPF as

$$s^5 + (2\zeta\omega + 2\zeta_c\omega_c)s^4 + \left(4\zeta\zeta_c\omega\omega_c + \omega^2 + \omega_c^2\right)s^3 + \left(2\zeta\omega\omega_c^2 + 2\zeta_c\omega_c\omega^2 + c_{11}^2 g_q\right)s^2 + \left[2\zeta_c\omega_c c_{11}^2 g_q - c_{11}^2\omega_c^2 g + \omega^2\omega_c^2\right]s + c_{11}^2 g_q\omega_c^2 = 0 \tag{10}$$

where s is complex variable in the complex s plane and c_{11} is a participation value for the first mode and one actuator/sensor pair. Because the feedback system now has an additional integrator, the augmented system is of fifth order. Equation (10) can be also generalized for the scalar system of any individual mode. The piezoceramic actuator is located at the first FEM element to maximize the control moment. If we chose appropriate g and g_q to make this system stable, then the steady-state disturbance as well as the vibrations would be significantly attenuated. The behavior of this system can be obtained from a root contour that traces the movement of the closed-loop poles as a function of g and g_q in Eq. (10). For the control of the approximated system, Fig. 4 shows the root contours of g_q, whereas g remains three fixed values. The circles show the root loci of the first mode for $g = 3000$ and $0 \leq g_q c_{11} \leq 50$, with an interval of 5, whereas squares and triangles are for $g = 20{,}000$ and 50,000, respectively. The participation value c_{11} is 0.0024 for

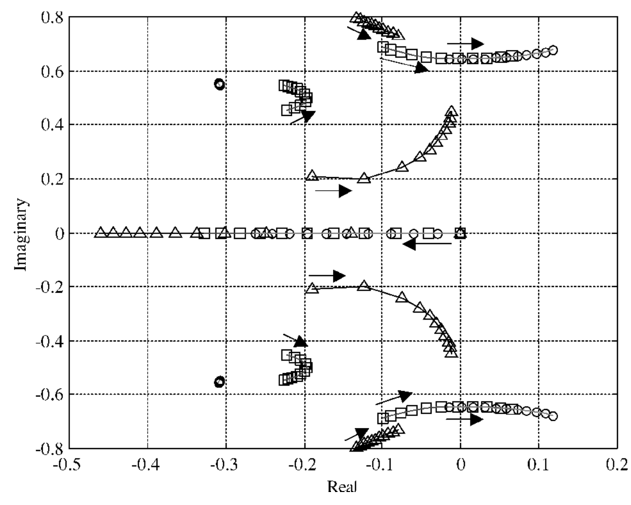

Fig. 4 Root contours of the boom system for the first mode control with the modified PPF: $g = 3000$ (○), $g = 20{,}000$ (□), $g = 50{,}000$ (△), $g_q c_{11}$ = 0–50 with interval of five.

the scalar system. From Fig. 4, it is indicated in certain regions that a larger g value yields a more stable system, and larger g_q forces the system to be less stable. Large values of the g coefficient of compensator increase the region of active PPF damping. The optimal damping coefficient of the compensator is dependent on how accurate the modes are known and how much they would be expected to change. The robustness of the system is also dependent on a compensator damping ratio. A damping coefficient of the compensator of about 0.5 leads to good damping and robustness. The compensator frequency is set at (or near) the first natural frequency of the boom, where it would give maximum efficiency. Good performance can be achieved by using the modified PPF, which yields a great suppression of the dominated structural disturbances, as shown in Figs. 2 and 3 for controlled cases with $g = 8000$ and $g_q = 843$. Figures 2 and 3 demonstrate that the attitude disturbances can be removed by the control moment from the smart structure using the modified PPF. We can also see that a single actuator at the base provides a sufficient degree of control when we use the modified PPF strategy. The unstable spillover effects on the uncontrolled second mode are shown. However, the excitation due to the unstable spillover effects is not significant because the excitation develops only unnoticeable attitude disturbances in the second mode, less than 0.01 deg for this special case. Different properties of the spacecraft and boom will yield different first mode frequencies. A decrease in the frequency of the first mode would reduce the control performance. The pole positions are very sensitive to control gain values. An increased robustness requires larger PPF control gains g and less integral control gains g_q. The same strategy of scalar control can be applied for other modes to suppress the vibration and magnitude of a chosen mode. The integral control of the n mode yields unstable spillover effects on the neighborhood modes, namely, the $n-1$ and $n+1$ modes.

Multimodes can be controlled simultaneously using the same number of compensators, one tuned to each mode. A root-contour technique provides a means of analyzing a bundled system containing only target modes, by varying one gain while the other remain fixed. For instance, the performance would have a better control system if the second mode as well as the first mode could be successfully controlled together. To control the first two modes with two compensators, one or two collocated actuator/sensor pair(s) can be used. For the system, the results using one actuator with two compensators are very similar to the results from two actuators with two compensators. In the two-modes control case, there are four independent gains that should be selected in the design process. The analyses of the multimodes characteristic equation show that the zeros are canceled with some poles because multimodes are included in the system, and the rest of poles are very close to the imaginary axis of the s plane. The integral control strengthens the pole-zero cancellations and induces instability in the system. Thus, we can expect the multimodes control system could be easily unstable, and its performance would not be good. Numerical simulations prove these anticipations from the root contours of the multimodes system. There are no improvements in performances, although a multimodes control skill is used to suppress the first and second modes simultaneously. This phenomenon implies that the good control regions are strictly limited, so that it provides sufficient performance in only the first mode control but not in multimodes. Fortunately, because most spacecraft have only a dominant first mode deformation due to thermal effects, the modified PPF of the scalar system can be a practical strategy to eliminate attitude disturbances caused by thermal loading.

Conclusions

A spacecraft with a flexible appendage is analyzed by a coupled system composed of a flexible boom and a rigid main body, which is described by the FEM. The thermally induced dynamic response of the boom shows a quasi-static displacement and superimposed vibrations, and, consequently, the attitude angle disturbances have a pointing error (steady-state error) as well as vibrations. To attenuate both the vibrations and constant attitude disturbances, an active control system combines the integral control with the original PPF, by using an embedded piezoceramic device as actuators and sensors. This modified PPF technique achieves great control performance when the compensator is tuned to the modal frequency of the system, although the implemented integral control makes the system complicated and enforces unstable spillover effects. The dynamics of a bundled subsystem, consisting of only the first mode, can well approximate the dynamics of the whole spacecraft system because the thermal loading excites mainly the first mode of the system. Thus, by using only a scalar control strategy, the modified PPF technique can provide good capability of controlling the main attitude disturbances from thermally induced structural deformations.

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